

Indian Statistical Institute, Bangalore

B. Math (II)

First semester 2012-2013

Mid-Semester Examination : Statistics (I)

Date: 12-09-2012

Maximum Score 75

Duration: 3 Hours

1. The data below show IQ scores for 30 sixth graders.

088	102	126	095	109	099
102	151	115	097	092	107
081	119	094	090	109	099
102	117	098	093	105	084
114	122	087	094	101	081

- (a) Make a stem and leaf plot of these data.
- (b) Find the sample mean \bar{X} .
- (c) Find the sample standard deviation S .
- (d) Find the sample median M .
- (e) Find 100 p -th percentile for $p = 0.25$ and $p = 0.75$.
- (f) Find the second quartile Q_2 .
- (g) What proportion of the data lies within $\bar{X} \pm 3S$?
- (h) Draw the box plot and identify the outliers.
- (i) Decide on trimming fraction just enough to eliminate the outliers and obtain the trimmed mean \bar{X}_T .
- (j) Also obtain the trimmed standard deviation S_T .
- (k) Between the box plot and the stem and leaf plot what do they tell us about the above data set? In very general terms what can you say about the population from which the data have arrived?

$$[4 + 2 + 3 + 2 + 4 + 2 + 2 + 5 + 3 + 4 + 4 = 35]$$

2. For random variables X and Y define the correlation coefficient ρ_{XY} . If the joint density function of the two random variables X and Y , for $a > 0$, $\lambda > 0$ and $b \in \mathbb{R}$, is given by

$$f(x, y|\lambda) = \frac{1}{a + \lambda}; \quad 0 < x < 1, \quad ax + b < y < ax + b + a + \lambda;$$

then find ρ_{XY} . Find $\lim_{\lambda \rightarrow 0} \rho_{XY}$ and $\lim_{\lambda \rightarrow \infty} \rho_{XY}$.

$$[3 + 10 + 2 = 15]$$

[PTO]

3. Let X_1, X_2, \dots, X_n be a random sample from the distribution with *pdf* given by

$$f(x|\theta_1, \theta_2) = \frac{1}{\theta_2 - \theta_1} I_{(\theta_1, \theta_2)}(x); \theta_1 < \theta_2 \in \mathbb{R}.$$

Find *maximum likelihood estimators (MLE)* for θ_1 and θ_2 .

[10]

4. Let Y be distributed as $\exp(\lambda)$, $\lambda > 0$. Define $X = \alpha e^Y$, $\alpha > 0$. Obtain the distribution F_X of X . Suppose we have a random sample X_1, X_2, \dots, X_n from F_X , $\alpha > 0$ and $\lambda > 0$ both unknown. Obtain *method of moments (MOM)* estimators for α and λ based on X_1, X_2, \dots, X_n . Also obtain *maximum likelihood estimators (MLE)* for α and λ .

[4 + 10 + 8 = 22]